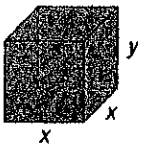
PRECALC WORD PROBLEMS SEC. 3.4

1) **Minimizing Surface Area** United Parcel Service has contracted you to design a closed box with a square base that has a volume of 5000 cubic inches. See the illustration.



(a) Find a function for the surface area of the box.
(b) Using a graphing utility, graph the function found in part (a).
(c) What is the minimum amount of cardboard that can be used to construct the box?
(d) What are the dimensions of the box that minimize the surface area?
(e) Why might UPS be interested in designing a box that minimizes the surface area?

To minimize the cost of material that is needed for box construction.

a.) $SA = 4xy + 2x^2$

$V = x^2y \rightarrow 5000 = x^2y \rightarrow \frac{5000}{x^2} = y$

$SA = 4x\left(\frac{5000}{x^2}\right) + 2x^2 \rightarrow \boxed{SA = \frac{20000}{x} + 2x^2}$

b.) graph on calc

c.) use min function on calc to get (21.54, 2784.95)

↳ 2784.95 in^2 minimum cost

d.) $x = 21.54$, sub into $y = \frac{5000}{x^2}$ to get $y = 21.54$

↳ Dimensions = 21.54" x 21.54" x 21.54"

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2) **Measuring the Stress of Materials** The stress in the material of a pipe subject to internal pressure varies jointly with the internal pressure and the internal diameter of the pipe and inversely with the thickness of the pipe. The stress is 100 pounds per square inch when the diameter is 5 inches, the thickness is 0.75 inch, and the internal pressure is 25 pounds per square inch. Find the stress when the internal pressure is 40 pounds per square inch if the diameter is 8 inches and the thickness is 0.50 inch.

$S = \frac{K \cdot pd}{t} \rightarrow 100 = \frac{K \cdot 25 \cdot 5}{0.75}$

$K = 0.6$

$S = \frac{0.6 \cdot pd}{t}$

$S = \frac{0.6 \cdot 40 \cdot 8}{0.50} = \boxed{384 \text{ psi}}$

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3) **Average Cost** In Problem 95, Exercise 3.2, the cost function C (in thousands of dollars) for manufacturing x Chevy Cavaliers was determined to be

$$C(x) = 0.2x^3 - 2.3x^2 + 14.3x + 10.2$$

Economists define the **average cost function** as

$$\overline{C}(x) = \frac{C(x)}{x}$$

(a) Find the average cost function.
(b) What is the average cost of producing six Cavaliers?
(c) What is the average cost of producing nine Cavaliers?
(d) Using a graphing utility, graph the average cost function.
(e) Using a graphing utility, find the number of Cavaliers that should be produced to minimize the average cost.
(f) What is the minimum average cost?

↳ $9400

a.) $\overline{C}(x) = \frac{C(x)}{x}$

$\boxed{\overline{C}(x) = \frac{0.2x^3 - 2.3x^2 + 14.3x + 10.2}{x}}$

b.) $\overline{C}(6) = \$9400$

c.) $\overline{C}(9) \approx \$10,933$

d.) graph $\overline{C}(x)$ on calculators

e.) use min function on CALC to get (6, 9400)

↳ 6 = # of Cavaliers that should be produced to minimize avg cost